

# Basic Genome-wide Association Studies

Robert Brown

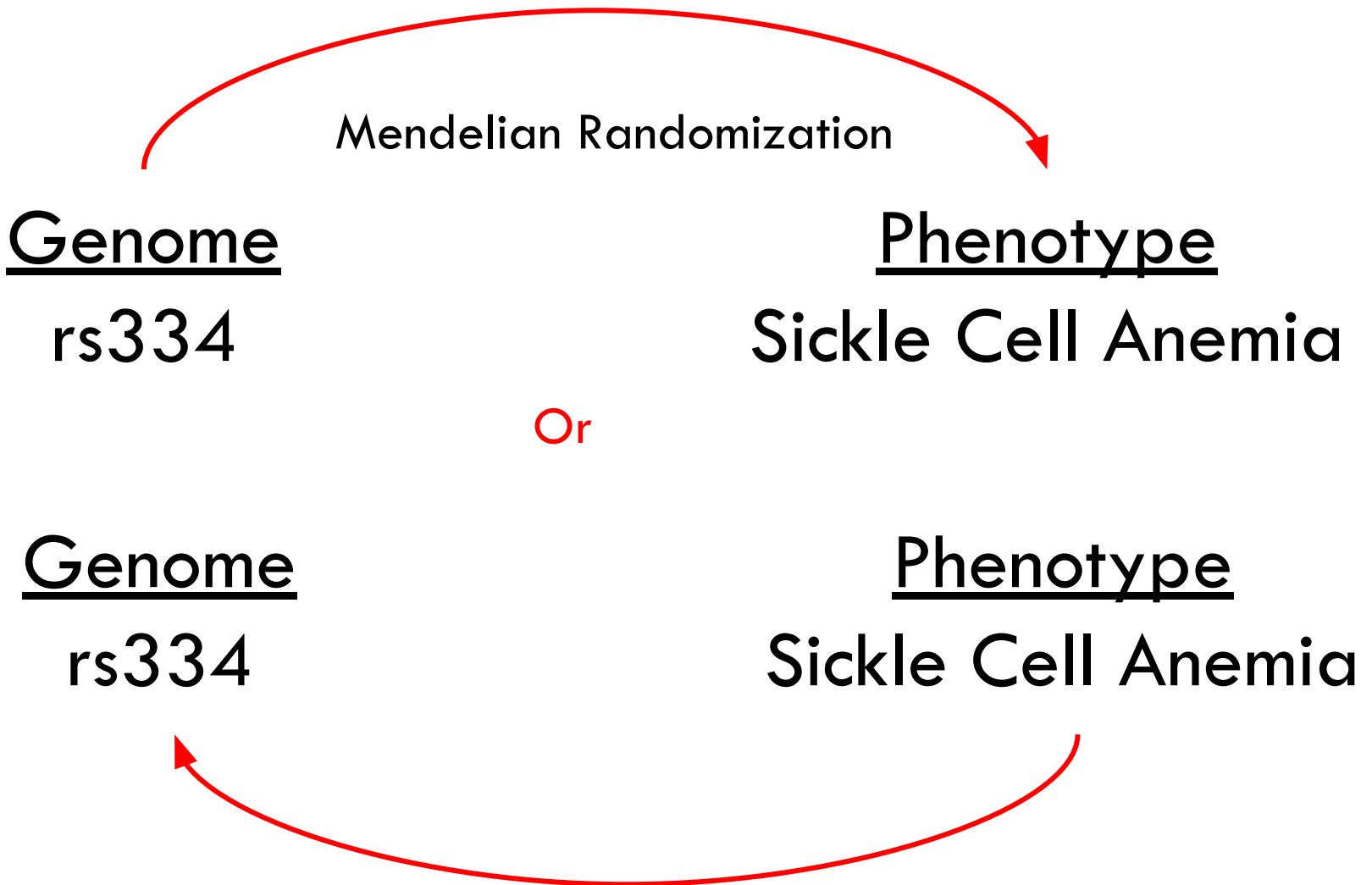
Thank you!

**UCLA** Computational Medicine

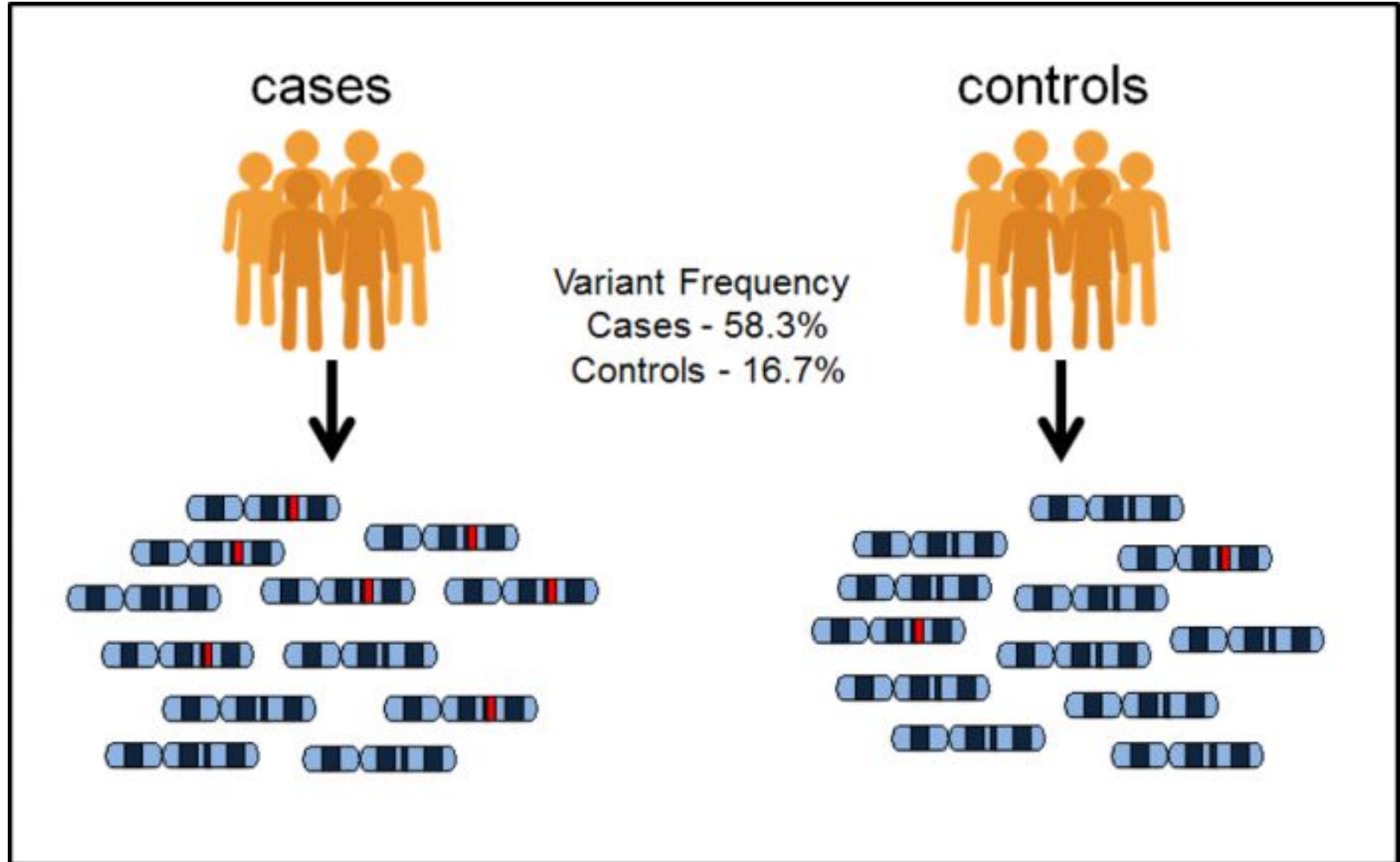
# What is a phenotype

- Height
- BMI
- hair color
- diabetes
- autism
- annual salary
- favorite color
- blood sugar level
- RELN protein level

# Sickle Cell Anemia: Causal Direction



# Genome-wide association study (GWAS) Case-Control



# Genome-wide association study (GWAS)

## Continuous Phenotype

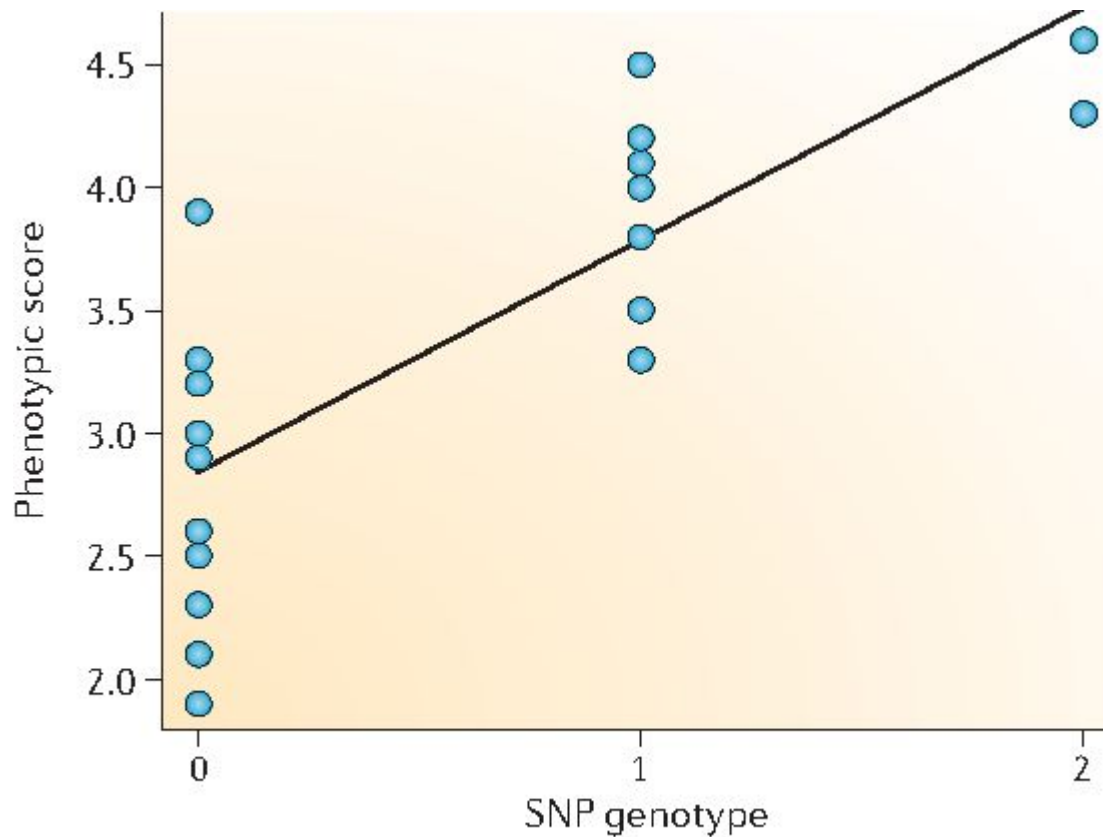


Figure 3 | Linear regression test of single-SNP

# GWAS Model: linear algebra style

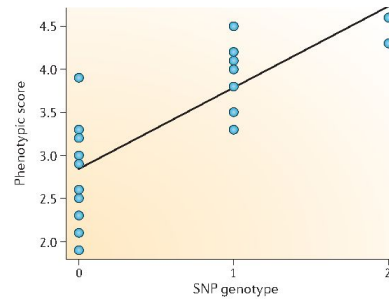


Figure 3 | Linear regression test of single-SNP

## Full Model

$$Y = G\beta + \varepsilon$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1M} \\ g_{21} & g_{22} & \cdots & g_{2M} \\ \vdots & \vdots & \vdots & \vdots \\ g_{N1} & g_{N1} & \cdots & g_{NM} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

## Marginal Model

$$Y = G_i\beta_i + \varepsilon$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} g_{1i} \\ g_{2i} \\ \vdots \\ g_{Ni} \end{bmatrix} \beta_i + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

Not enough power to “fit” this model

# GWAS Model

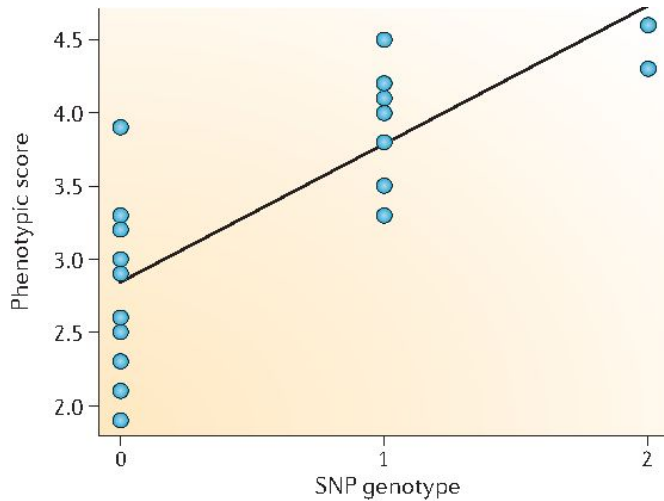


Figure 3 | Linear regression test of single-SNP

$$y_j = \beta_i g_{ij} + \varepsilon_j$$

individual:  $j$

SNP:  $i$

phenotype:  $y$

genotype:  $g$

Effect of SNP  $i$ :  $\beta$

all other effects:  $\varepsilon$

$$y_j = \beta_{Sex} S_j + \beta_{Age} A_j + \beta_{PC1} PC_{1j} + \beta_{PC2} PC_{2j} + \beta_i g_{ij} + \varepsilon_j$$

$\beta_i$ : true effect

$\hat{\beta}_i$ : estimated effect



# GWAS Model

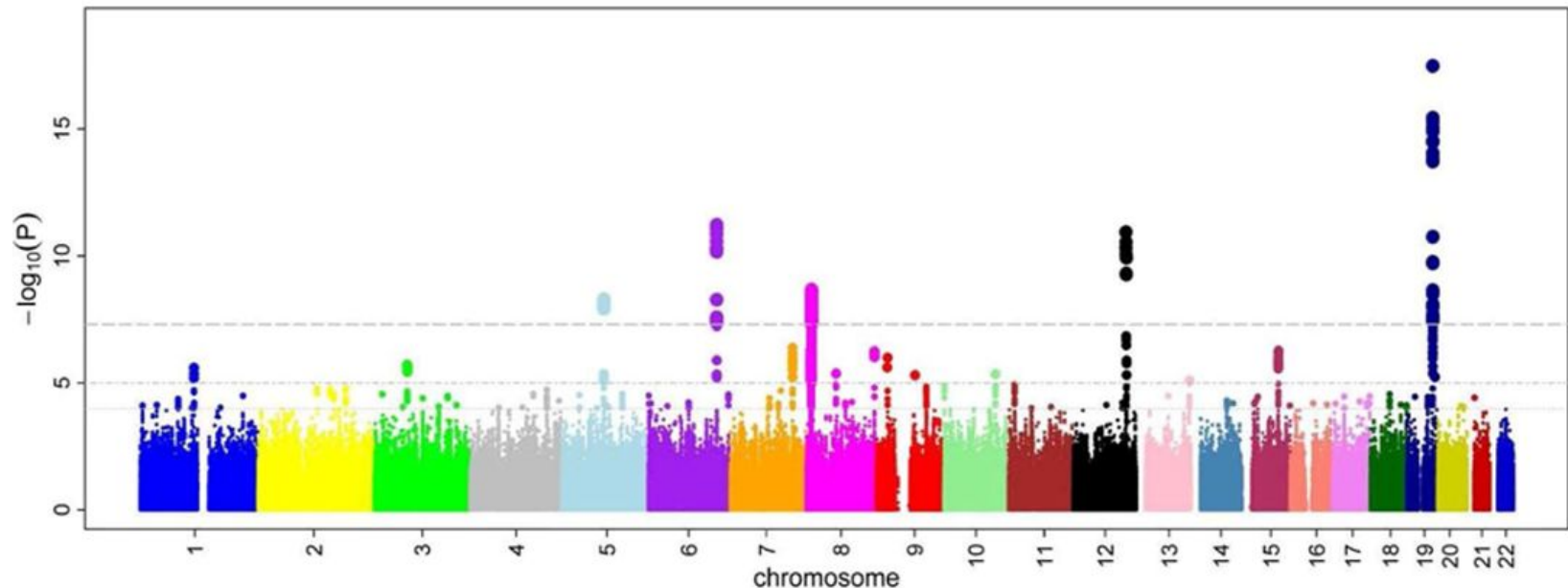
## Marginal Model

$$Y = G_i \beta_i + \varepsilon$$

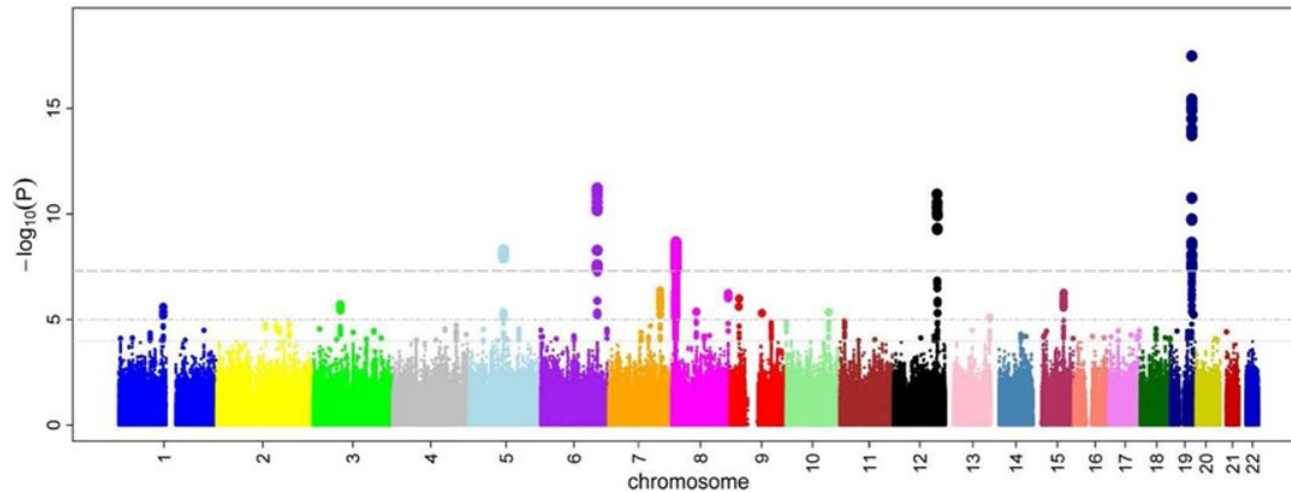
Null: genotype has no estimated effect

Alternate: genotype has non-zero estimated affect

$\hat{\beta}_i$ : estimated effect



# GWAS Model: P-values and FWER



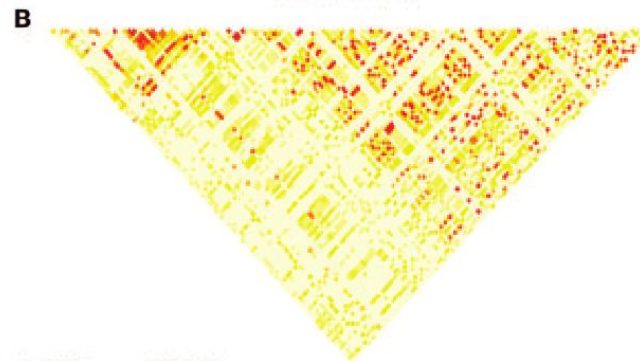
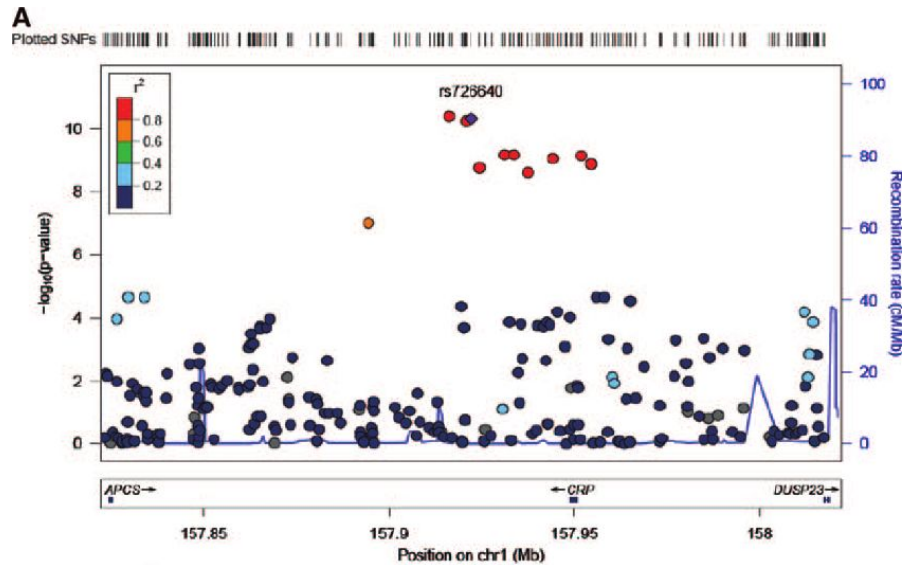
$$-\log_{10}(0.05) = 1.3$$

Family-wise error rate (FWER): The probability of have one or more false discoveries.

$$-\log_{10}(5 \times 10^{-8}) = 7.3$$

The above significance threshold controls FWER at a 0.05 rate in GWAS

# GWAS locus vs GWAS gene



Physical Distance: 119.3 kb

LD Map Type:  $r^2$



# GWAS: Summary

- Mendelian Randomization: phenotypes cannot affect genotypes
- GWAS “top hits” tag causal/risk variation, but it is not clear what is causal/risk variation
- GWAS finds a locus of the genome associated with a phenotype
- GWAS output: Estimated effects for each variant and the corresponding p-value
- FWER is controlled using a very small significance threshold